Continue

Look up hyperbolic in Wiktionary, the free dictionary. Hyperbolic may refer to: of or pertaining to a hyperbola, a type of smooth curve lying in a plane in mathematics Hyperbolic geometry, a non-Euclidean geometry Hyperbolic functions, analogues of ordinary trigonometric functions, defined using the hyperbola of or pertaining to hyperbole, the use of exaggeration as a rhetorical device or figure of speech Hyperbolic (album), by Pnau, 2024 All pages with titles beginning with Hyperbolic All pages with titles containing Hyperbolic Exaggeration Hyperboloid Topics referred to by the same term This disambiguation page lists articles associated with the title Hyperbolic.If an internal link led you here, you may wish to change the link to point directly to the intended article. Retrieved from " Home>Hyperbolic functions – Graphs, Properties, and ExamplesSolutionUse exponential form of $\cosh x$, $\sinh x = \dfrac{e^x + e^{-x}}{2}$ to evaluate the three expressions. Substitute $f(x) = \dfrac{e^x + e^{-x}}{2}$ with $x =0$, $x = \ln 2$, and $x = -\ln 2$.We've summarized the calculations in one table for you.\begin{aligned}\boldsymbol{f(0) }\end{aligned}\begin{aligned}\boldsymbol{f(\ln 2)}\end{aligned}\begin{aligned}\boldsymbol{f(-\ln 2)}\end{aligned}\begin{aligned}f(0)&= \dfrac{e^{0} + e^{-0}}{2}\\&= \dfrac{1 + 1}{2}\\&= 1\end{aligned}\begin{aligned}f(\ln 2)&= \dfrac{e^{\ln 2} + e^{-\ln 2}}{2}\\&=\dfrac{e^{\ln 2} + e^{\ln \frac{1}{2}}}{2},\\&= \dfrac{2 +\dfrac{1}{2}}{2}\color{Teal}e^{\ln a} =a\\&= \dfrac{5}{4}\end{aligned}\begin{aligned}f(\ln 2)&= \dfrac{e^{-\ln 2} + e^{-(-\ln 2)}}{2}\\&=\dfrac{e^{\ln \frac{1}{2}} + e^{\ln 2}}{2},\\&= \dfrac{\dfrac{1}{2} + 2}{2}\color{Teal}e^{\ln a} =a\\&= \dfrac{5}{4}\end{aligned}Hence, we have the following values for $f(x)$:a. $f(0) = 2$b. $f(\ln 2) = \dfrac{5}{4}$c. $f(-\ln 2) = \dfrac{5}{4}$Example 2Prove the hyperbolic trigonometric identities that are shown below.a.$1 – \text{tanh }^2 x= \text{sech }^2x$b. $\sinh 2x = 2\sinh x \cosh x$c. $\cosh (M + N) = \cosh M \cosh N + \sinh M \sinh N$SolutionLet's focus on rewriting $1 – \text{tanh }^2 x$ to $\text{sech }^2 x$. Begin by using the exponential form of $\tanh x = \dfrac{e^x – e^{-x}}{e^x + e^{-x}}$ to rewrite the left-hand side expression of the first equation.\begin{aligned}1 – \text{tanh }^2 x &= 1 – \left(\dfrac{e^x – e^{-x}}{e^x + e^{-x}} \right)^2\\&=1 – \left(\dfrac{e^{2x} – 2e^{-x}e^x + e^{-2x}}{e^{2x} + 2e^{-x}e^x + e^{-2x}} \right)\\&=1 – \left(\dfrac{e^{2x} – 2 + e^{-2x}}{e^{2x} + 2 + e^{-2x}} \right)\\&=\left(\dfrac{e^{2x} + 2 + e^{-2x}}{e^{2x} + 2 + e^{-2x}} \right) – \left(\dfrac{e^{2x} – 2 + e^{-2x}}{e^{2x} + 2 + e^{-2x}} \right)\\&= \dfrac{e^{2x} + 2 + e^{-2x}-e^{2x} + 2 – e^{-2x}}{e^{2x} + 2 + e^{-2x}}\\&= \dfrac{4}{e^{2x} + 2 + e^{-2x}} \end{aligned}Express the denominator as the square of $(e^x + e^{-x})$ then use the exponential form of $\text{sech }x = \dfrac{2}{e^x + e^{-x}}$ to rewrite the resulting expression.\begin{aligned}1 – \text{tanh }^2 x &= \dfrac{4}{(e^{x} + e^{-x})^2}\\&= \left(\dfrac{2}{e^{x} + e^{-x}} \right)^2\\&= (\text{sech }x)^2\\&= \text{sech }^2x \end{aligned}a. Hence, we've shown that $1 – \text{tanh }^2 x= \text{sech }^2x$.Moving on to the second identity, we'll start by using the exponential form of $\sinh x$ and $\cosh x$ to rewrite the right-hand side of the equation.\begin{aligned}2\sinh x \cosh x &= 2\left(\dfrac{e^{x} – e^{-x}}{2} \right)\left(\dfrac{e^{x} + e^{-x}}{2} \right)\\&= 2\left(\dfrac{e^{2x} – e^{-2x}}{4} \right),\color{Teal}(a-b)(a +b) = a^2 -b^2 \\&= \dfrac{e^{2x} – e^{-2x}}{2}\\&= \sinh 2x\end{aligned}b. This confirms that $\sinh 2x = 2\sinh x \cosh x$.For the third identity, let's manipulate the right-hand side of the expression. Again, we use the exponential forms of $\sinh M$, $\cosh M$, $\sinh N$, and $\cosh N$ to rewrite the expression. Begin by simplifying $\cosh M \cosh N$ and $\sinh M \sinh N$ first separately.\begin{aligned}\boldsymbol{\cosh M \cosh N }\end{aligned}\begin{aligned}\boldsymbol{\sinh M \sinh N }\end{aligned}\begin{aligned}\cosh M \cosh N &= \left(\dfrac{e^M + e^{-M}}{2} \right)\left(\dfrac{e^N + e^{-N}}{2} \right)\\&= \dfrac{e^Me^N + e^Me^{-N} + e^{-M}e^N + e^{-M}e^{-N}}{4}\end{aligned}\begin{aligned}\sinh M \sinh N &= \left(\dfrac{e^M – e^{-M}}{2} \right)\left(\dfrac{e^N – e^{-N}}{2} \right)\\&= \dfrac{e^Me^N – e^Me^{-N} – e^{-M}e^N + e^{-M}e^{-N}}{4}\end{aligned}\begin{aligned}\cosh M \cosh N + \sinh M \sinh N &= \dfrac{2e^Me^N + 2e^{-M}e^{-N}}{4} \end{aligned}Simplify the sum of $\cosh M \cosh N$ and $\sinh M \sinh N $ further to come up with the left-hand side of the identity.\begin{aligned}\cosh M \cosh N + \sinh M \sinh N &= \dfrac{e^Me^N + e^{-M}e^{-N}}{2}\\&= \dfrac{e^{M + N} + e^{-(M + N)}}{2}\\&= \cosh(M + N) \end{aligned}c. Therefore, $\cosh (M + N) = \cosh M \cosh N + \sinh M \sinh N$.Example 3Evaluate the derivative of the following hyperbolic functions:a. $\dfrac{d}{dx}\cosh (x – n)$b. $\dfrac{d}{dx} \tanh (\ln x)$SolutionUse the derivative rule for $\cosh x$ and apply the chain rule to differentiate $\cosh(x –n)$ as shown below.\begin{aligned}\dfrac{d}{dx} \cosh (x – n) &= {\color{Teal}\sinh(x -n)} \cdot {\color{Orchid}\dfrac{d}{dx}(x -n)},{\color{Teal}\dfrac{d}{dx}\cosh x=\sinh x}\text{ & }{\color{Orchid}\text{Chain Rule}}\\&= \sinh (x -n)\cdot \left({\color{Teal}\dfrac{d}{dx}x - \dfrac{d}{dx}n}\right),{\color{Teal}\text{Difference Rule}}\\&= \sinh (x -n)\cdot ({\color{Teal}1} - {\color{Orchid}0}),{\color{Teal}\text{Power Rule}}\text{ & }{\color{Orchid}\text{Constant Rule}}\\&=\sinh (x -n) \end{aligned}a. This means that $\dfrac{d}{dx} \cosh (x – n) = \sinh (x –n)$.Apply the derivative rule, $\dfrac{d}{dx} \tanh x = \text{sech }^2 x$ to differentiate the second expression. Make sure to apply the chain rule for the inner function.\begin{aligned}\dfrac{d}{dx} \tanh(\ln x) &= {\color{Teal}\text{sech }^2x(\ln x)} \cdot {\color{Orchid}\dfrac{d}{dx}(\ln x)},{\color{Teal}\dfrac{d}{dx}\tanh x=\text{sech }^2x}\text{ & }{\color{Orchid}\text{Chain Rule}}\\&= \text{sech }^2(\ln x)\cdot \left({\color{Teal}\dfrac{1}{x}}\right),{\color{Teal}\dfrac{d}{dx}\ln x = \dfrac{1}{x}}\\&= \dfrac{1}{x}\text{sech }^2{(\ln x)} \end{aligned}Simplify $\text{sech }^2(\ln x)$ further by using the exponential form of $\text{sech }x$.\begin{aligned}\dfrac{1}{x}\text{sech }^2{(\ln x)} &= \dfrac{1}{x}\cdot\left(\dfrac{2}{{\color{Teal} x} + {\color{Teal} \dfrac{1}{x}} } \right)^2 ,\color{Teal} e^{\ln a} =a\\&= \dfrac{1}{x}\left(\dfrac{2}{\dfrac{x^2 + 1}{x}} \right)^2\\&= \dfrac{4x}{(x^2 + 1)^2}\end{aligned}b. Hence, we've shown that $\dfrac{d}{dx} \tanh (\ln x)$ is equal to $\dfrac{1}{x}\text{sech }^2 (\ln x)$ or $\dfrac{4x}{(x^2 + 1)^2}$.Example 4Let's say we have $\sinh x = \dfrac{5}{13}$, what are the exact values of x that satisfy the equation?SolutionFind the value of $\cosh x$ using the identity, $\cosh^2 x =1 + \sinh^2x$, and the fact that $\sinh x = \dfrac{5}{12}$.\begin{aligned}\cosh^2 x =1 + \sinh^2 x\\\cosh^2 x &= 1 + \left(\dfrac{5}{12}\right)^2 \\\cosh^2 x&= \dfrac{169}{144}\\\cosh x &= \pm \dfrac{13}{12}\end{aligned}Since $\cosh x$ is always positive, we'll use $\cosh x = \dfrac{13}{12}$. Now, let's observe what happens when we add $\sinh x$ and $\cosh x$.\begin{aligned}\sinh x &= \dfrac{e^x – e^{-x}}{2}\\\cosh x &= \dfrac{e^x + e^{-x}}{2}\\\\\sinh x + \cosh x &= \dfrac{2e^x}{2}\\&= e^x\end{aligned}This means that $\dfrac{5}{12} + \dfrac{13}{12} = e^x$. Simplify the right-hand side then take the natural logarithm of both sides of the equation to find x.\begin{aligned}\dfrac{18}{12}&= e^x\\\ln\left(\dfrac{3}{2}\right)&= \ln(e^x),{\color{Teal}\ln(e^a) =a}\\x&= \ln\left(\dfrac{3}{2} \right)\end{aligned}This means that the exact value of x is $\ln\left(\dfrac{3}{2}\right)$.Practice Questions1. Given that $f(x) = \sinh x$. Evaluate the values of the following expressions without using a calculator:a. $f(0)$b. $f(\ln 4)$c. $f(-\ln 4)$2. Given that $g(x) = \tanh x$. Evaluate the values of the following expressions without using a calculator:a. $f(0)$b. $f(\ln 5)$c. $f(-\ln 5)$3. Prove the hyperbolic trigonometric identities that are shown below.a.$\cosh 2x = 2\cosh^2 x -1$b. $\sinh (M – N) = \sinh M \cosh N – \sinh N \cosh M$4. Show that $\dfrac{d}{dx} \text{tanh }x = \text{sech }^2 x$.5. Evaluate the derivative of the following hyperbolic functions:a. $\dfrac{d}{dx} \tanh (x – n)$b. $\dfrac{d}{dx} \cosh (\ln x)$c. $\dfrac{d}{dx} 3x^2 \sinh x$6. Let's say we have $\cosh x = \dfrac{41}{40}$, what are the exact values of x that satisfy the equation?Answer Key1. a.$f(0) =0$b. $f(\ln 4) = \dfrac{15}{8}$c. $f(-\ln 4) =- \dfrac{15}{8}$2.a.$g(0) =0$b. $g(\ln 5) = \dfrac{12}{13}$c. $g(-\ln 5) =- \dfrac{12}{13}$3.a.$\begin{aligned}2\cosh^2 x -1&= 2\left(\dfrac{e^x + e^{-x}}{2} \right)^2- 1\\&= \dfrac{e^{2x} +2 + e^{-2x}}{2}-1\\&=\dfrac{e^{2x} +2+ e^{-2x} -2}{2}\\&= \dfrac{e^{2x} + e^{-2x}}{2}\\&= \cosh 2x\end{aligned}b.$\begin{aligned}\sinh M \cosh N &= \left(\dfrac{e^M – e^{-M}}{2} \right)\left(\dfrac{e^N + e^{-N}}{2} \right)\\&=\dfrac{e^M e^N +e^M e^{-N}-e^{-M}e^N -e^{-M}e^{-N}}{4} \\ \sinh M \cosh M&= \left(\dfrac{e^M – e^{-M}}{2} \right)\left(\dfrac{e^M + e^{-M}}{2} \right)\\&=\dfrac{e^M e^N +e^M e^{-N}-e^{-M}e^N -e^{-M}e^{-N}}{4}\\\\\sinh M \cosh N-\sinh N \cosh&= \dfrac{2e^{M -N} + 2e^{-(M -N)}}{4}\&= \dfrac{e^{M -N} – e^{-(M-N)}}{2}\\&= \sinh (M -N)\end{aligned}$4.$\begin{aligned}\dfrac{d}{dx} \text{tanh }x &= \dfrac{d}{dx}\left(\dfrac{e^x-e^{-x}}{e^x + e^{-x}}\right)\\&= \dfrac{(e^x + e^{-x})\dfrac{d}{dx}(e^x-e^{-x})-(e^x – e^{-x})\dfrac{d}{dx}(e^x+e^{-x})}{(e^x + e^{-x})^2}\\&= \dfrac{(e^x + e^{-x})(e^x + e^{-x})-(e^x – e^{-x})(e^x – e^{-x})}{(e^x + e^{-x})^2}\\&= \dfrac{4}{(e^x + e^{-x})^2}\\&= \left(\dfrac{2}{e^x + e^{-x}}\right)^2\\&=\text{sech }^2x\end{aligned}$5.a. $\dfrac{d}{dx} \tanh (x – n) = \text{sech }^2 (x –n)$b. $\dfrac{d}{dx} \cosh (\ln x) = \dfrac{1}{x} \sinh(\ln x)= \dfrac{x^2 -1}{2x}$ c. $\dfrac{d}{dx} 3x^2 \sinh x = 3(2x \sinh x + x^2 \cosh x)$6. $x = \ln\left(\dfrac{5}{4}\right)$Images/mathematical drawings are created with GeoGebra. Hyperbolic functions are similar to trigonometric functions, but instead of unit circles, they are defined using rectangular hyperbolas. In trigonometry, the coordinates on a unit circle are represented as (cos θ, sin θ), whereas in hyperbolic functions, the pair (cosh θ, sinh θ) represents points on the right half of an equilateral hyperbola. They are used in solving linear differential equations, hyperbolic geometry, and Laplace's equations in Cartesian coordinates. The three basic hyperbolic functions are: Hyperbolic sine (sinh) Hyperbolic cosine (cosh) Hyperbolic tangent (tanh) Hyperbolic functions are expressed through exponential function ex and its inverse e-x (here, e = Euler's constant). The hyperbolic sine function is a function f: ℝ → ℝ such that f(x) = sinh x, which is expressed as sinh x = ${\dfrac{e^{x}-e^{-x}}{2}}$ Now, let us graph the function y = sinh x and see how it behaves. The hyperbolic cosine function is a function f: ℝ → ℝ such that f(x) = cosh x, which is expressed as cosh x = ${\dfrac{e^{x}+e^{-x}}{2}}$ Now, on graphing the hyperbolic cosine function y = cosh x, we get: The hyperbolic tangent function is a function f: ℝ → ℝ such that f(x) = tanh x, which is expressed as tanh x = ${\dfrac{e^{x}-e^{-x}}{e^{x}+e^{-x}}}$ Now, on graphing the hyperbolic tangent function y = tanh x, we get In addition to the three basic hyperbolic functions, there are three other hyperbolic functions equivalent to sine, cosine, and tangent. They are secant, cosecant, and cotangent: The hyperbolic cosecant function (denoted by cosech or csch) is the reciprocal of the hyperbolic sine function, which is given by the expression: cosech x = ${\dfrac{1}{\sinh x}}$ = ${\dfrac{2}{e^{x}-e^{-x}}}$ Now, on graphing the hyperbolic cosecant function y = cosech x, we get Similarly, the hyperbolic secant function (sech) is the reciprocal of the hyperbolic cosine function, which is given by the expression: sech x = ${\dfrac{1}{\cosh x}}$ = ${\dfrac{2}{e^{x}+e^{-x}}}$ Now, on graphing the hyperbolic secant function y = sech x, we get The hyperbolic cotangent function (coth) is the reciprocal of the hyperbolic tangent function, which is given by the expression: coth x = ${\dfrac{1}{\tanh x}}$ = ${\dfrac{e^{x}+e^{-x}}{e^{x}-e^{-x}}}$ Now, on graphing the hyperbolic cotangent function y = coth x, we get Here is the summary of all graphs of the hyperbolic functions: The domain and range of the six hyperbolic functions are listed: Hyperbolic FunctionDomainRangesinh x(-∞, ∞)(-∞, ∞)cosh x(-∞, ∞)[1, ∞)tanh x(-∞, ∞)(-1, 1)cosech x(-∞, 0) U (0, ∞)(-∞, 0) U (0, ∞)sech x(-∞, ∞)(0, 1]coth x(-∞, 0) U (0, ∞)(-∞, – 1) U (1, ∞) Hyperbolic identities, similar to those of trigonometric functions, are related to the relationships between hyperbolic functions. These are universally true for all values of the variables involved. cosh2 (x) – sinh2 (x) = 1 tanh2 (x) + sech2 (x) = 1 coth2 (x) – cosech2 (x) = 1 sinh (-x) = -sinh (x) cosh (−x) = cosh (x) tanh (−x) = −tanh (x) coth (-x) = -coth x sech (-x) = sech x cosec (-x) = -cosech x sinh x + sinh y = ${2\sinh \left(\dfrac{x+y}{2}\right) \cosh \left(\dfrac{x-y}{2}\right)}$ sinh x – sinh y = ${2\cosh \left(\dfrac{x+y}{2}\right) \sinh \left(\dfrac{x-y}{2}\right)}$ cosh x + cosh y = ${2\cosh \left(\dfrac{x+y}{2}\right) \cosh \left(\dfrac{x-y}{2}\right)}$ cosh x – cosh y = ${2\sinh \left(\dfrac{x+y}{2}\right) \sinh \left(\dfrac{x-y}{2}\right)}$ 2 sinh x cosh y = sinh (x + y) + sinh (x -y) 2 cosh x sinh y = sinh (x + y) – sinh (x – y) 2 sinh x sinh y = cosh (x + y) – cosh (x – y) 2 cosh x cosh y = cosh (x + y) + cosh (x – y) sinh (x ± y) = sinh x cosh x ± coshx sinh y cosh (x ± y) = cosh x cosh y ± sinh x sinh y tanh (x ± y) = ${\dfrac{\tanh x\pm \tanh y}{1\pm \tanh x\tanh y}}$ coth (x ± y) = ${\dfrac{\coth x\coth y\pm 1}{\coth y\pm \coth x}}$ cosh 2x = 1 + 2 sinh2 x = 2 cosh2 x – 1 cosh 2x = cosh2 x + sinh2 x sinh 2x = 2 · sinh x · cosh x The derivatives of hyperbolic functions are similar to those of trigonometric functions. ${\dfrac{d}{dx}\left(\sinh x\right) =\cosh x}$ ${\dfrac{d}{dx}\left(\cosh x\right) =\sinh x}$ ${\dfrac{d}{dx}\left(\tanh x\right) =sech ^{2}x}$ ${\dfrac{d}{dx}\left(cosech \ x\right) =-cosech \ x\cdot \coth x}$ ${\dfrac{d}{dx}\left(sech \ x\right) =-sech \ x\cdot \tanh x}$ ${\dfrac{d}{dx}\left(coth x\right) =-cosech ^{2}x}$ The inverse hyperbolic functions are the inverse operations of the hyperbolic functions. These functions are also known as area hyperbolic functions that provide the hyperbolic angles corresponding to a given value of the hyperbolic function. Here is the list of all inverse hyperbolic functions in the complex plane: sinh-1 x = ${\ln \left(x+\sqrt{x^{2}-1}\right)}$ cosh-1 x = ${\ln \left(x+\sqrt{x^{2}-1}\right)}$ tanh-1 x = ${\dfrac{1}{2}\ln \left(\dfrac{1+x}{1-x}\right)}$ cosech-1 x = ${\ln \left(\dfrac{1+\sqrt{1+x^{2}}}{x}\right)}$ sech-1 x = ${\ln \left(\dfrac{1+\sqrt{1-x^{2}}}{x}\right)}$ coth-1 x = ${\dfrac{1}{2}\ln \left(\dfrac{x+1}{x-1}\right)}$ Using hyperbolic functions identities, prove:a) cosh x + sinh x = exb) cosh2 x – sinh2 x = 1c) coth2x – cosech2x = 1Solution:a) As we know,sinh x = ${\dfrac{e^{x}-e^{-x}}{2}}$ and cosh x = ${\dfrac{e^{x}+e^{-x}}{2}}$Here,cosh x + sinh x= ${\dfrac{e^{x}-e^{-x}}{2}}$+${\dfrac{e^{x}+e^{-x}}{2}}$= ${\dfrac{e^{x}-e^{-x}+e^{x}+e^{-x}}{2}}$= ${\dfrac{2e^{x}}{2}}$= ${e^{x}}$Hence provedb) As we know,sinh x = ${\dfrac{e^{x}-e^{-x}}{2}}$ and cosh x = ${\dfrac{e^{x}+e^{-x}}{2}}$Here,cosh2 x – sinh2 x= ${\left(\dfrac{e^{x}+e^{-x}}{2}\right) ^{2}-\left(\dfrac{e^{x}-e^{-x}}{2}\right) ^{2}}$= ${\dfrac{\left(e^{x}+e^{-x}\right) ^{2}-\left(e^{x}-e^{-x}\right) ^{2}}{4}}$= ${\dfrac{4\cdot e^{x}\cdot e^{-x}}{4}}$= ${e^{x-x}}$= ${e^{0}}$= 1Hence provedc) As we know,cosh x = ${\dfrac{\cosh x}{\sinh x}}$cosech x = ${\dfrac{1}{\sinh x}}$Here,coth2x – cosech2x= ${\left(\dfrac{\cosh x}{\sinh x}\right) ^{2}-\left(\dfrac{1}{\sinh x}\right) ^{2}}$= ${\dfrac{\cosh ^{2}x}{\sinh ^{2}x}-\dfrac{1}{\sinh ^{2}x}}$= ${\dfrac{\cosh ^{2}x-1}{\sinh ^{2}x}}$ .....(i)Since cosh2 x – sinh2 x = 1⇒ cosh2 x – 1 = sinh2 x .....(ii)From (i) and (ii),coth2x – cosech2x = ${\dfrac{\sinh ^{2}x}{\sinh ^{2}x}}$ = 1Hence proved Last modified on November 25th, 2024 Hyperbolic Functions are similar to trigonometric functions but their graphs represent the rectangular hyperbola. These functions are defined using hyperbola instead of unit circles. Hyperbolic functions are expressed in terms of exponential functions ex. In this article, we will learn about the hyperbolic function in detail, including its definition, formula, and graphs. Table of Content The six basic hyperbolic functions are, Hyperbolic sine or sinh xHyperbolic cosine or cosh xHyperbolic tangent or tanh xHyperbolic cosecant or cosech xHyperbolic secant or sech xHyperbolic cotangent or coth xHyperbolic functions are defined using exponential functions. They are represented as, sinh x which is read as hyperbolic sinh x. Then the sinh x is defined as, sinh x = (ex + e-x)/2 Similarly, other hyperbolic functions are defined. Hyperbolic Functions FormulasVarious hyperbolic function formulas are, sinh(x) = (ex - e-x)/2 FunctionDefinitionHyperbolic Cosine (cosh x)cosh(x) = (ex + e-x)/2Hyperbolic Sine (sinh x)Hyperbolic Tangent (tanh x)tanh(x) = sinhx/coshx = (ex - e-x)/(ex + e-x)Hyperbolic Cotangent (coth x)coth(x) = cosh x/sin hx = (ex + e-x)/(ex - e-x)Hyperbolic Secant (sech x) ech(x) = 1/cosh x = 2/(ex + e-x)Hyperbolic Cosecant (csch x) csch(x) = 1/sinh x = 2/(ex - e-x)Domain and Range of Hyperbolic FunctionsDomain and Range are the input and output of a function, respectively. The domain and range of various hyperbolic functions are added in the table below: Hyperbolic FunctionDomainRangesinh x(-∞, +∞)(-∞, +∞)cosh x(-∞, +∞)[1, ∞)tanh x(-∞, +∞)(-1, 1)coth x(-∞, 0) U (0, +∞)(-∞, -1) U (1, + ∞)sech x(-∞, + ∞)(0, 1]csch x(-∞, 0) U (0, + ∞)(-∞, 0) U (0, + ∞)Learn, Domain and Range of a Function Properties of Hyperbolic FunctionsVarious properties of the hyperbolic functions are added below, sinh (-x) = – sinh(x)cosh (-x) = cosh (x)tanh (-x) = - tanh xcoth (-x) = - coth xsech (-x) = sech xcsc (-x) = - csch xcosh 2x = 1 + 2 sinh2(x) = 2 cosh2x - 1cosh 2x = cosh2x + sinh2xsinh 2x = 2 sinh x cosh xHyperbolic functions are also derived from trigonometric functions using complex arguments. Such that, sinh x = - i sin(ix)cosh x = cos(ix)tanh x = - i tan(ix)coth x = i cot(ix)sech x = sec(ix)Hyperbolic Trigonometric IdentitiesThere are various identities that are related to hyperbolic functions. Some of the important hyperbolic trigonometric identities are, sinh(x ± y) = sinh x cosh y ± coshx sinh ycosh(x ± y) = cosh x cosh y ± sinh x sinh ytanh(x ± y) = (tanh x ± tanh y)/ (1± tanh x tanh y)coth(x ± y) = (coth x coth y ± 1)/(coth y ± coth x)sinh x – sinh y = 2 cosh [(x+y)/2] sinh [(x-y)/2]sinh x + sinh y = 2 sinh [(x+y)/2] cosh[(x-y)/2]cosh x + cosh y = 2 cosh [(x+y)/2] cosh[(x-y)/2]cosh x – cosh y = 2 sinh [(x+y)/2] sinh [(x-y)/2]cosh2x - sinh2x = 1tanh2x + sech2x = 1coth2x - csch2x = 12 sinh x cosh (x + y) + sinh (x - y)2 cosh x sinh y = sinh (x + y) – sinh (x - y)2 sinh x sinh y = cosh (x + y) – cosh (x – y)2 cosh x cosh y = cosh (x + y) + cosh (x – y)Also, Check Trigonometric Identities Hyperbolic Functions DerivativeDerivatives of Hyperbolic functions are used to solve various mathematical problems. The derivative of hyperbolic cos x is hyperbolic sin x, i.e. d/dx (cosh x) = sinh x = d(cosh(x))/dx = d((ex + e-x)/2)/dx = 1/2(d(ex + e-x)/dx) = 1/2(ex - e-x) = sinh(x) Similarly, derivatives of other hyperbolic functions are found. The table added below shows the hyperbolic functions. Derivatives of Hyperbolic Functions Hyperbolic FunctionDerivativesinh xcosh xcosh xsinh xtanh xsech2 xcoth x-csch2 xsech x-sech x.tanh xcsch x-csch x.coth xLearn, Derivative in Maths Integration of Hyperbolic FunctionsIntegral Hyperbolic functions are used to solve various mathematical problems. The integral of hyperbolic cos x is hyperbolic sin x, i.e. ∫ (cosh x).dx = sinh x + C The table added below shows the integration of various hyperbolic functions. Integral of Hyperbolic Functions Hyperbolic FunctionIntegralsinh xcosh x + Ccosh xsinh x + Ctanh xln (cosh x) + Ccoth xln (sinh x) + Csech xarctan (sinh x) + Ccsch xln (tanh(x/2)) + CLearn, Integration Inverse Hyperbolic FunctionsInverse hyperbolic functions are found by taking the inverse of the hyperbolic function, i.e. if y = sinh x then, x = sinh-1 (y) this represents the inverse hyperbolic sin function. Now the inverse of various hyperbolic function are, sinh-1x = ln (x + √(x2 + 1))cosh-1x = ln (x + √(x2 - 1))tanh-1x = ln [(1 + x)/(1 - x)]coth-1x = ln [(x + 1)/(x - 1)]sech-1x = ln [{1 + √(1 - x2)}/x]csch-1x = ln [{1 + √(1 + x2)}/x] Learn More: Inverse Trigonometric IdentitiesHyperbolic Functions ExamplesExample 1: Find the value of x solving, 4sinh x - 6cosh x - 2 = 0. Solution: We know that, sinh x = (ex - e-x)/2cosh x = (ex + e-x)/2Given, 4sinh x - 6cosh x + 2 = 0 ⇒ 4[(ex - e-x)/2] - 6[(ex + e-x)/2] + 6 = 0 ⇒ 2(ex - e-x) - 3(ex + e-x) + 6 = 0 ⇒ 2ex - 2e-x - 3ex - 3e-x + 6 = 0 ⇒ -ex - 5e-x + 6 = 0 ⇒ -e2x - 5 + 6ex = 0 ⇒ e2x - 6ex + 5 = 0 ⇒ e2x - 5ex - ex + 5 = 0 ⇒ ex(ex - 5) - 1(ex - 5) = 0 ⇒ (ex - 1)(ex - 5) = 0 (ex - 1) = 0 ex = 1 x = 0 (ex - 5) = 0 ex = 5 x = ln 5 Example 2: Prove, cosh x + sinh x = ex Solution: LHS = cosh x + sinh x = (ex - e-x)/2 + (ex + e-x)/2 = (ex - e-x + ex + e-x)/2 = 2ex / 2 = ex = RHS Hyperbolic Functions Practice QuestionsQ1: Find the value of x solving, sinh x + 5cosh x - 4 = 0 Q2: Find the value of x solving, 2sinh x - 6cosh x - 5 = 0 Q3: Find the value of x solving, 9sinh x + 6cosh x + 11 = 0 Q4: Find the value of x solving, sinh x - cosh x - 3 = 0 Related : The two basic hyperbolic functions are "sinh" and "cosh": Hyperbolic Sine: sinh(x) = ex − e-x 2 (pronounced "shine") Hyperbolic Cosine: cosh(x) = ex + e-x 2 (pronounced "cosh") They use the natural exponential function ex And are not the same as sin(x) and cos(x), but a little bit similar: sinh vs sin cosh vs cos Catenary One of the interesting uses of Hyperbolic Functions is the curve made by suspended cables or chains. A hanging cable forms a curve called a catenary defined using the cosh function: f(x) = a cosh(x/a) Like in this example from the page arc length : Other Hyperbolic Functions From sinh and cosh we can create: Hyperbolic tangent "tanh" (pronounced "than"): tanh(x) = sinh(x) cosh(x) = ex − e-x ex + e-x tanh vs tan Hyperbolic cotangent: coth(x) = cosh(x) sinh(x) = ex + e-x ex − e-x Hyperbolic secant: sech(x) = 1 cosh(x) = 2 ex + e-x Hyperbolic cosecant "csch" or "cosech": csch(x) = 1 sinh(x) = 2 ex − e-x Why the Word "Hyperbolic" ? Because it comes from measurements made on a Hyperbola: So, just like the trigonometric functions relate to a circle, the hyperbolic functions relate to a hyperbola. Identities sinh(−x) = −sinh(x) cosh(−x) = cosh(x) And tanh(−x) = −tanh(x) coth(−x) = −coth(x) sech(−x) = sech(x) csch(−x) = −csch(x) Odd and Even Both cosh and sech are Even Functions, the rest are Odd Functions. Derivatives The Derivatives are: d dx sinh(x) = cosh(x) d dx cosh(x) = sinh(x) d dx tanh(x) = 1 − tanh2(x) Copyright © 2023 Rod Pierce