The binomial distribution is a commonly used discrete distribution in statistics. The normal distribution as opposed to a binomial distribution is a continuous distribution. The binomial distribution represents the probability for 'x' successes of an experiment in 'n' trials, given a success probability 'p' for each trial at the experiment. Binomial Distribution in Statistics: The binomial distribution forms the base for the famous binomial test of statistical importance. A test that has a single outcome such as success/failure is also called a Bernoulli trial or Bernoulli experiment, and a series of outcomes is called a Bernoulli process. Consider an experiment where each time a question is asked for a yes/no with a series of n experiments. Then in the binomial probability distribution, the boolean-valued outcome the success/yes/true/one is represented with probability p and the failure/no/false/zero with probability q (q = 1 − p). In a single experiment when n = 1, the binomial distribution is called a Bernoulli distribution. What Is the Binomial Distribution Formula? The binomial distribution formula is for any random variable X, given by; P(x:n,p) = nC\(_x\) px (1-p)n-x Or P(x:n,p) = nCx px (q)n-x where, n = the number of experiments x = 0, 1, 2, 3, 4, … p = Probability of success in a single experiment q = Probability of failure in a single experiment (= 1 – p) The binomial distribution formula is also written in the form of n-Bernoulli trials, where nCx = n!/x!(n-x)!. Hence, P(x:n,p) = n!/[x!(n-x)!].px.(q)n-x Want to find complex math solutions within seconds? Use our free online calculator to solve challenging questions. With Cuemath, find solutions in simple and easy steps. Book a Free Trial Class Examples on Binomial Distribution Formula Example 1: If a coin is tossed 5 times, using binomial distribution find the probability of: (a) Exactly 2 heads (b) At least 4 heads. Solution: (a) The repeated tossing of the coin is an example of a Bernoulli trial. According to the problem: Number of trials: n=5 Probability of head: p= 1/2 and hence the probability of tail, q =1/2 For exactly two heads: x=2 P(x=2) = 5C2 p2 q5-2 = 5! / 2! 3! × (½)2× (½)3 P(x=2) = 5/16 (b) For at least four heads, x ≥ 4, P(x ≥ 4) = P(x = 4) + P(x=5) Hence, P(x = 4) = 5C4 p4 q5-4 = 5!/4! 1! × (½)4× (½)1 = 5/32 P(x = 5) = 5C5 p5 q5-5 = (½)5 = 1/32 Answer: Therefore, P(x ≥ 4) = 5/32 + 1/32 = 6/32 = 3/16 Example 2: For the same question given above, find the probability of getting at most 2 heads. Solution: Solution: P(at most 2 heads) = P(X ≤ 2) = P (X = 0) + P (X = 1) P(X = 0) = (½)5 = 1/32 P(X=1) = 5C1 (½)5.= 5/32 Answer: Therefore, P(X ≤ 2) = 1/32 + 5/32 = 3/16 Example 3: 60% of people who purchase sports cars are men. Find the probability that exactly 7 are men if 10 sports car owners are randomly selected. Solution: Let's Identify 'n' and 'X' from the problem. The number of sports car owners are randomly selected is n = 10, and The number to find the probability is X = 7. Given: p = 60%, or 0.6. Therefore, the probability of failure is q = 1 – 0.6 = 0.4 Now, using the binomial distribution formula \(P(x) = \frac{{n!}}{{({n - x})!x!}}.(p)^x .(q)^{n - x} \\ = \frac{{10!}}{{({10 - 7})!7!}}.({0.6})^7 .t({0.4})^{10 - 7} \\ = 120 \times 0.0279936 \times 0.064 \\ = 0.215 \) Answer: The probability that exactly 7 are men is 0.215 or 21.5%. The binomial distribution is a common discrete distribution used in statistics, as opposed to a continuous distribution, such as the normal distribution. The binomial distribution, therefore, represents the probability for x successes in n trials, given a success probability p for each trial. The binomial distribution formula is for any random variable X, given by; P(x:n,p) = nC\(_x\) px (1-p)n-x Or P(x:n,p) = nC\(_x\) px (q)n-x, where, n is the number of experiments, p is probability of success in a single experiment, q is probability of failure in a single experiment (= 1 – p) and takes values as 0, 1, 2, 3, 4, …, n. What Is the Purpose of the Binomial Distribution Formula? The binomial distribution formula allows us to compute the probability of observing a specified number of "successes" when the process is repeated a specific number of times (e.g., in a set of patients) and the outcome for a given patient is either a success or a failure. What Is the Formula for Binomial Distribution? The formula for binomial distribution is: P(x: n,p) = nC\(_x\) px (q)n-x Where p is the probability of success, q is the probability of failure, n = number of trials. What Is the Binomial Distribution Formula for the Mean and Variance? The mean and variance of the binomial distribution are: Mean = np Variance = npq where p is the probability of success, q is the probability of failure, n = number of trials. The binomial distribution is one of the most popular distributions in statistics. To understand the binomial distribution, it helps to first understand binomial experiments. A binomial experiment is an experiment that has the following properties: The experiment consists of n repeated trials. Each trial has only two possible outcomes. The probability of success, denoted p, is the same for each trial. Each trial is independent. The most obvious example of a binomial experiment is a coin flip. For example, suppose we flip a coin 10 times. This is a binomial experiment because it has the following four properties: The experiment consists of n repeated trials – There are 10 trials. Each trial has only two possible outcomes – heads or tails. The probability of success, denoted p, is the same for each trial – If we define "success" as landing on heads, then the probability of success is exactly 0.5 for each trial. Each trial is independent – The outcome of one coin flip does not affect the outcome of any other coin flip. The binomial distribution describes the probability of obtaining k successes in n binomial experiments. If a random variable X follows a binomial distribution, then the probability that X = k successes can be found by the following formula: P(X=k) = nCk * pk * (1-p)n-k where: n: number of trials k: number of successes p: probability of success on a given trial nCk: the number of ways to obtain k successes in n trials For example, suppose we flip a coin 3 times. We can use the formula above to determine the probability of obtaining 0, 1, 2, and 3 heads during these 3 flips: P(X=0) = 3C0 * .50 * (1-.5)3-0 = 1 * 1 * (.5)3 = 0.125 P(X=1) = 3C1 * .51 * (1-.5)3-1 = 3 * .5 * (.5)2 = 0.375 P(X=2) = 3C2 * .52 * (1-.5)3-2 = 3 * .25 * (.5)1 = 0.375 P(X=3) = 3C3 * .53 * (1-.5)3-3 = 1 * .125 * (.5)0 = 0.125 We can create a simple histogram to visualize this probability distribution: It's straightforward to calculate a single binomial probability (e.g. the probability of a coin landing on heads 1 time out of 3 flips) using the formula above, but to calculate cumulative binomial probabilities we need to add individual probabilities. For example, suppose we want to know the probability that a coin lands on heads 1 time or less out of 3 flips. We would use the following formula to calculate this probability: P(X≤1) = P(X=0) + P(X=1) = 0.125 + 0.375 = 0.5. This is known as a cumulative probability because it involves adding more than one probability. We can calculate the cumulative probability of obtaining k or less heads for each outcome using a similar formula: P(X≤0) = P(X=0) = 0.125. P(X≤1) = P(X=0) + P(X=1) = 0.125 + 0.375 = 0.5. P(X≤2) = P(X=0) + P(X=1) + P(X=2) = 0.125 + 0.375 + 0.375 = 0.875. P(X≤3) = P(X=0) + P(X=1) + P(X=2) + P(X=3) = 0.125 + 0.375 + 0.375 + 0.125 = 1. We can create a histogram to visualize this cumulative probability distribution: When we're working with small numbers (e.g. 3 coin flips), it's reasonable to calculate binomial probabilities by hand. However, when we're working with larger numbers (e.g. 100 coin flips), it can be cumbersome to calculate probabilities by hand. In these cases, it can be helpful to use a binomial probability calculator like the one below. For example, suppose we flip a coin n = 100 times, the probability that it lands on heads in a given trial is p = 0.5, and we want to know the probability that it will land on heads k = 43 times or less: P(X=43) = 0.03007 P(X43) = 0.90333 P(X≥43) = 0.93339 Here is how to interpret the output: The probability that the coin lands on heads exactly 43 times is 0.03007. The probability that the coin lands on heads less than 43 times is 0.06661. The probability that the coin lands on heads 43 times or less is 0.09667. The probability that the coin lands on heads more than 43 times is 0.90333. The probability that the coin lands on heads 43 times or more is 0.93339. The binomial distribution has the following properties: The mean of the distribution is μ = np The variance of the distribution is σ2 = np(1-p) The standard deviation of the distribution is σ = √np(1-p) For example, suppose we toss a coin 3 times. Let p = the probability the coin lands on heads. The mean number of heads we would expect is μ = np = 3*.5 = 1.5. The variance in the number of heads we would expect is σ2 = np(1-p) = 3*.5*(1-.5) = 0.75. Use the following practice problems to test your knowledge of the binomial distribution. Problem 1 Question: Bob makes 60% of his free-throw attempts. If he shoots 12 free throws, what is the probability that he makes exactly 10? Answer: Using the Binomial Distribution Calculator above with p = 0.6, n = 12, and k = 10, we find that P(X=10) = 0.06385. Problem 2 Question: Jessica flips a coin 5 times. What is the probability that the coin lands on heads 2 times or fewer? Answer: Using the Binomial Distribution Calculator above with p = 0.5, n = 5, and k = 2, we find that P(X≤2) = 0.5. Problem 3 Question: The probability that a given student gets accepted to a certain college is 0.2. If 10 students apply, what is the probability that more than 4 get accepted? Answer: Using the Binomial Distribution Calculator above with p = 0.2, n = 10, and k = 4, we find that P(X>4) = 0.03279. Problem 4 Question: You flip a coin 12 times. What is the mean expected number of heads that will show up? Answer: Recall that the mean of a binomial distribution is calculated as μ = np. Thus, μ = 12*0.5 = 6 heads. Problem 5 Question: Mark hits a home run during 10% of his attempts. If he has 5 attempts in a given game, what is the variance of the number of home runs he'll hit? Answer: Recall that the variance of a binomial distribution is calculated as σ2 = np(1-p). Thus, σ2 = 6*.1*(1-.1) = 0.54. The following articles can help you learn how to work with the binomial distribution in different statistical softwares: Binomial Distribution is a probability distribution used to model the number of successes in a fixed number of independent trials, where each trial has only two possible outcomes: success or failure. This distribution is useful for calculating the probability of a specific number of successes in scenarios like flipping coins, quality control, or survey predictions.Binomial Distribution is based on Bernoulli trials, where each trial has an independent and identical chance of success. The probability distribution for a Bernoulli trial is called the Bernoulli Distribution.A Binomial Distribution for a random variable X = 0, 1, 2,…, n is defined as the probability of success or failure in a series of independent trials. Each trial is independent of the others, and the distribution helps calculate the probability of various outcomes in these trials.Conditions for Binomial Distribution The Binomial distribution can be used in scenarios where the following conditions are satisfied:Fixed Number of Trials: There are a set number of trials or experiments (denoted by n), such as flipping a coin 10 times.Two Possible Outcomes: Each trial has only two possible outcomes, often labeled as "success" and "failure." For example, getting heads or tails in a coin flip.Independent Trials: The outcome of each trial is independent of the others, meaning the result of one trial does not affect the result of another.Constant Probability: The probability of success (denoted by p) remains the same for each trial. For example, if you're flipping a fair coin, the probability of getting heads is always 0.5.The Binomial distribution is an appropriate model to use for calculating the probabilities of obtaining a certain number of successes in the given trials.Read More: Bernoulli trailsNegative Binomial DistributionThe Negative Binomial Distribution is used to model the number of trials needed to achieve a certain number of successes in a sequence of independent trials, where the probability of success in each trial is constant.For example, consider a situation where getting 6 is the success of throwing a die. Now if we throw the die and not get 6 then it is a failure. Now we throw again and do not get 6. Let's say we don't get 6 for three successive attempts and 6 is obtained in the fourth attempt and onwards then the binomial distribution of the number of getting 6 is called the Negative Binomial Distribution.Negative Binomial Distribution FormulaThe formula for Negative Binomial Distribution is given asP(x) = n+r-1Cr-1prqnWhere,n = Total Number of Trials.r = Number of Trials in which we get the first success.p = Probability of Success in Each Trial.q = Probability of Failure in Each Trial.Binomial Distribution FormulaThe Binomial Distribution Formula which is used to calculate the probability, for a random variable X = 0, 1, 2, 3,….,n is given asP(X = r) = nCrprqn-r, r = 0, 1, 2, 3….Where, n = Total number of trialsr = Number of successesp = Probability of successq = Probability of failure (q = 1 - p)Bernoulli Trials in Binomial DistributionBernoulli Trial is a trial that gives results of dichotomous nature i.e. result in yes or no, head or tail, even or odd. It means it gives two types of outcomes out of which one favors the event while the other doesn't. A random experiment is called Bernoulli Trial if it satisfies the following conditions:Trials are finite in numberTrials are independent of each otherEach trial has only two possible outcomesThe probability of success and failure in each trial is the same.Binomial Random VariableA Binomial Random Variable can be defined by two possible outcomes such as "success" and binomial "failure". For instance, consider rolling a fair six-sided die and recording the value of the face. The binomial distribution formula can be put into use to calculate the probability of success for binomial distributions. Often it states "plugin" the numbers to the formula and calculates the requisite values.The binomial distribution is based on the following characteristics:Experiment contains n identical trials.Each trial results in one of the two outcomes either success or failure.The probability of success, denoted p, remains the same from trial to trial.All the n trials are independent.Example: A fair coin is flipped 20 times; X represents the number of headsX is a binomial random variable with n = 20 which is the total number of trials and p = 1/2 is the probability of getting head in each trial. The value of X represents the number of trials in which you succeed in getting head.Binomial Distribution CalculationBinomial Distribution in statistics is used to compute the probability of likelihood of an event using the above formula. To calculate the probability using binomial distribution we need to follow the following steps:Step 1: Find the number of trials and assign it as 'n'Step 2: Find the probability of success in each trial and assign it as 'p'Step 3: Find the probability of failure and assign it as q where q = 1-pStep 4: Find the random variable X = r for which we have to calculate the binomial distributionStep 5: Calculate the probability of Binomial Distribution for X = r using the Binomial Distribution Formula.The use of the above steps has been illustrated using an example below:Binomial Distribution ExamplesFinding the probability of getting exactly 6 heads when a fair coin is flipped 10 times.Finding the probability of exactly 3 bulbs being defective when a batch of 100 bulbs is tested and each bulb has a 2% chance of being defective.To find the Probability of exactly 7 patients responding positively to the treatment when the drug is tested on 8 patients and has a 90% success rate.Let's say we toss a coin twice, and getting head is a success we have to calculate the probability of success and failure then, in this case, we will calculate the probability distribution as follows:In each trial getting a head that is a success, its probability is given as:p = 1/2n = 2 as we throw a coin twicer = 0 for no success, r = 1 for getting head once and r = 2 for getting head twiceProbability of failure q = 1 - p = 1 - 1/2 = 1/2.P(Getting 1 head) = P(X = 1) = ncrprqn-r = 2c1 (1/2)1(1/2)1 = 2 × 1/2 × 1/2 = 1/2P(Getting 2 heads) = P(X = 2) = 2c2(1/2)2(1/2)0 = 1/4P(Getting 0 heads) = P(X = 0) = 2c0(1/2)0(1/2)2 = 1/4Random Variable (X = r)P(X = r)X = 0 (Getting 0 Head)1/4X = 1 (Getting 1 Head)1/2X = 2 (Getting 2 Head)1/4As of now, we know that Binomial Distribution is calculated for the Random Variables obtained in Bernoulli Trials. Hence, we should understand these terms.Binomial Distribution TableThe binomial distribution for a situation when getting 6 is a success on throwing two dies is discussed in this section. First of all, we see that it is a Bernoulli Trial as getting 6 is the only success, and getting any different is a failure. Now we can get six on both die in a trial or six on only one of the die in a trial and getting no six on both die. Hence, the random variable for which we have to find the probability takes the value X = r = 0, 1, 2. The Binomial Distribution Table for getting 6 as success is plotted below:Random Variable (X = r)P(X = r)X = 0 (Getting no 6)25/36X = 1 (Getting one 6)10/36X = 2 (Getting two 6)1/36We see that sum of all the probabilities 25/36 + 10/36 + 1/36 = 1.Binomial Distribution GraphBinomial Distribution Graph is plotted for X and P(X). We will plot a Binomial Distribution Graph for tossing a coin twice where getting the head is a success. If we toss a coin twice, the possible outcomes are {HH, HT, TH, TT}. The binomial distribution Table for this is given below:X (Random Variable)P(X)X = 0 (Getting no head)P(X = 0) = 1/4 = 0.25X = 1 (Getting 1 head)P(X = 1) = 2/4 = 1/2 = 0.5X = 2 (Getting two heads)P(X = 2) = 1/4 = 0.25Binomial Distribution Graph for the above table is given below:Binomial Distribution in StatisticsMeasures of central tendency, specifically the mean, provide insights into the distribution's central or typical value for the number of successes in a series of independent trials. For a binomial distribution defined by parameters n (number of trials) and p (probability of success on each trial), the measures of central tendency are characterized as follows:Binomial Distribution MeanBinomial Distribution VarianceBinomial Distribution Standard DeviationMeasure of Central Tendency for Binomial DistributionThe formulas for Mean, Variance, and Standard Deviation of Binomial Distribution are listed below:Binomial Distribution MeanThe Mean of Binomial Distribution is the measurement of average success that would be obtained in the 'n' number of trials. The Mean of Binomial Distribution is also called Binomial Distribution Expectation. The formula for Binomial Distribution Expectation is given as:μ = n.pwhere,μ is the Mean or Expectationn is the Total Number of Trialsp is the Probability of Success in Each TrialRead more about, Expected Value or ExpectationExample: If we toss a coin 20 times and getting head is the success then what is the mean of success?Solution:Total Number of Trials n = 20Probability of getting head in each trial, p = 1/2 = 0.5Mean = n.p = 20 × 0.5It means on average we would head 10 times on tossing a coin 20 times.Binomial Distribution VarianceVariance of Binomial Distribution tells about the dispersion or spread of the distribution. It is given by the product of the number of trials, probability of success, and probability of failure. The formula for Variance is given as follows:σ2 = n.p.qwhereσ2 is Variancen is the Total Number of Trialsp is the Probability of Success in Each Trialq is the Probability of Failure in Each TrialExample: If we toss a coin 20 times and getting head is the success then what is the variance of the distribution?Solution:We have, n = 20Probability of Success in each trial (p) = 0.5Probability of Failure in each trial (q) = 0.5Variance of the Binomial Distribution, σ = n.p.q = (20 × 0.5 × 0.5) = 5 Binomial Distribution Standard DeviationStandard Deviation of Binomial Distribution tells about the deviation of the data from the mean. Mathematically, Standard Deviation is the square root of the variance. The formula for the Standard Deviation of Binomial Distribution is given asσ = √n.p.qwhere,σ is the Standard Deviationn is the Total Number of Trialsp is the Probability of Success in Each Trialq is the Probability of Failure in Each TrialExample: If we toss a coin 20 times and getting head is the success then what is the standard deviation?Solution:We have, n = 20Probability of Success in each trial (p) = 0.5Probability of Failure in each trial (q) = 0.5Standard Deviation of the Binomial Distribution, σ = √n.p.q ⇒ σ = √(20 × 0.5 × 0.5) ⇒ σ = √5 = 2.23Binomial Distribution PropertiesProperties of Binomial Distribution are mentioned below:There are only two possible outcomes: success or failure, yes or no, true or false.There is a finite number of trials given as 'n'.The probability of success and failure in each trial is the same.Only Success is calculated out of all trials.Each trial is independent of any other trial.Binomial Distribution Applications Binomial Distribution is used where we have only two possible outcomes. Let's see some of the areas where Binomial Distribution can be used.To find the number of male and female students in an institute.To find the likeability of something in Yes or No.To find defective or good products manufactured in a factor.To find positive and negative reviews on a product.Votes are collected in the form of 0 or 1.Binomial Distribution vs Normal DistributionBinomial Distribution differs from the Normal Distribution in many aspects. The key differences and characteristics of the Binomial and Normal distributions are highlighted in the following table: AspectBinomial DistributionNormal DistributionTypeDiscrete probability distributionContinuous probability distributionOutcomesTwo possible outcomes per trial (success or failure)Infinite possible outcomes within a continuous rangeParametersn (number of trials), p (probability of success)μ (mean), σ (standard deviation)ShapeVaries depending on n and p; typically skewed unless p=0.5 and n is largeBell-shaped curve (symmetric)Supportx can take integer values from 0 to nx can take any real number (from −∞ to +∞)Meanμ = npμVariance2 = np(1 -p)2ApplicabilityUsed for modeling the number of successes in a fixed number of independent trialsUsed for modeling continuous data that cluster around a meanExamplesFlipping coins, quality control (defective items)Heights of people, test scores, measurement errorsApproximationApproximates Normal distribution for large n and p not too close to 0 or 1Considered the limit of the Binomial Distribution as n becomes large and p is near 0.5People Also Read:Binomial Distribution in Probability ExamplesExample 1: A die is thrown 6 times and if getting an even number is a success what is the probability of getting (i) 4 Successes (ii) No successSolution:Given: n = 6, p = 3/6 = 1/2, and q = 1 - 1/2 = 1/2P(X = r) = nCrprqn-r(i) P(X = 4) = 6C4(1/2)4(1/2)2 = 15/64(ii) P(X = 0) = 6C0(1/2)0(1/2)6 = 1/64Example 2: A coin is tossed 4 times what is the probability of getting at least 2 heads? Solution:Given: n = 4Probability of getting head in each trial, p = 1/2 ⇒ q = 1 - 1/2 = 1/2P(X = r) = 4Cr(1/2)r(1/2)4-r⇒ P(X = r) = 4Cr(1/2)4 {Using the laws of Exaponents}And we know, Probability of getting at least 2 heads = P(X ≥ 2) ⇒ Probability of getting at least 2 heads = P(X = 2) + P(X = 3) + P(X = 4)⇒ Probability of getting at least 2 heads = 4C2(1/2)4 + 4C3(1/2)4 + 4C4(1/2)4⇒ Probability of getting at least 2 heads = (4C2 + 4C3 + 4C4)(1/2)4⇒ Probability of getting at least 2 heads = 11(1/2)4 = 11/16Example 3: A pair of dice is thrown 6 times and getting sum 5 is a success then what is the probability of getting (i) no success (ii) two success (iii) at most two successSolution:Given: n = 65 can be obtained in 4 ways (1, 4) (4, 1) (2, 3) (3, 2)Probability of getting the sum 5 in each trial, p = 4/36 = 1/9Probability of not getting sum 5 = 1 - 1/9 = 8/9(i) Probability of getting no success, P(X = 0) = 6C0(1/9)0(8/9)6 = (8/9)6(ii) Probability of getting two success, P(X = 2) = 6C2(1/9)2(8/9)4 = 15(84/96)(iii) Probability of getting at most two successes, P(X ≤ 2) = P(X = 0) + P(X = 1) + P(X = 2)⇒ P(X ≤ 2) = (8/9)6 + 6(85/96) + 15(84/96)Practice Problems on Binomial Distribution in Probability1. A box has 5 red, 7 black,? and 8 white balls. If three balls are drawn one by one with replacement what is the probability that all,i) all are white ii) all are red iii) all are black2. What is the probability distribution of the number of tails when three coins are tossed together?3. A die is thrown three times what is the probability distribution of getting six?4. A coin is tossed 4 times then what is the probability distribution of getting head. In statistics and probability theory, the binomial distribution is the probability distribution that is discrete and applicable to events having only two possible results in an experiment, either success or failure. (the prefix "bi" means two, or twice). A few circumstances where we have binomial experiments are tossing a coin: head or tail, the result of a test: pass or fail, selected in an interview: yes/ no, or nature of the product: defective/non-defective. Such a distribution of a binomial random variable is called a binomial probability distribution. Binomial Distribution is a commonly used discrete distribution in statistics. The normal distribution as opposed to a binomial distribution is a continuous distribution. Let us learn the formula to calculate the Binomial distribution considering many experiments and a few solved examples for a better understanding. What Is Binomial Distribution? The binomial distribution is the probability distribution of a binomial random variable. A random variable is a real-valued function whose domain is the sample space of a random experiment. Let us consider an example to understand this better. Toss a fair coin twice. This is a binomial experiment. There are 4 possible outcomes of this experiment. {HH, HT, TH, TT}. Consider getting one head as the success. Count the number of successes in each possible outcome. Here n(getting heads) is the success in n repeated trials of a binomial experiment.. n(X) = 0, 1, or 2 is the binomial random variable. The distribution of probability is of a binomial random variable, and this is known as a binomial distribution. No. of heads(n(X)) Probability of getting a head(P(X)) 0 P(x = 0) = 1/4 = 0.25 1 P(x = 1) = P(HT) = 1/4 + 1/4 = 0.50 2 P(x = 2) = P(HH) = 1/4 = 0.25 This table shows that getting one head in a single flip is 0.50. Now if a coin is flipped 3 times, consider we are intended to find the binomial distribution of getting two heads. Tossing 3 coins result in 8 outcomes. {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}. The probability of getting two heads [P(HH)] is 3/8. Similarly, we can calculate the probability of getting one head, 2 heads, and 3 heads and 0 heads. The binomial probability distribution is given in terms of a random variable as: P(X = 0) = 1/8 P(X = 1) = 3/8 P(X = 2) = 3/8 P(X = 3)= 1/8 Binomial Distribution in Statistics The binomial distribution forms the base for the famous binomial test of statistical importance. The binomial distribution represents the probability for 'x' successes of an experiment in 'n' trials, given a success probability 'p' for each trial at the experiment. Two parameters n and p are used here in the binomial distribution. The variable 'n' represents the number of trials and the variable 'p' states the probability of any one(each) outcome. A test that has a single outcome such as success/failure is also called a Bernoulli trial or Bernoulli experiment, and a series of outcomes is called a Bernoulli process. Consider an experiment where each time a question is asked for a yes/no with a series of n experiments. Then in the binomial probability distribution, the boolean-valued outcome the success/yes/true/one is represented with probability p and the failure/no/false/zero with probability q (q = 1 − p). In a single experiment when n = 1, the binomial distribution is called a Bernoulli distribution. If a die is thrown randomly 10 times, then the probability of getting a 3 for any throw is 1/6. Similarly, if we throw the dice 10 times, we have n = 10 and p = 1/6, q = 5/6 Negative Binomial Distribution Let's understand with an example when can a binomial distribution be negative. Suppose we throw a die and determine that the occurrence of 2 will be a failure and all non-2's will be successes. Let the failures be denoted by 'r'. Now, if the die is thrown frequently until 2 appears the third time, i.e., r = three failures, then the binomial distribution of the number of non-2's that arrived would be the negative binomial distribution. Binomial Distribution Formula The binomial distribution formula is for any random variable X, given by; P(x:n,p) = nCx px (1-p)n-x Or P(x:n,p) = nCx px (q)n-x Where p is the probability of success, q is the probability of failure, and n = number of trials. The binomial distribution formula is also written in the form of n-Bernoulli trials. where nCx = n!/x!(n-x)!. Hence, P(x:n,p) = n!/[x!(n-x)!].px.(q)n-x Binomial Distribution Calculation The image given below shows the formula used for binomial distribution calculation: Application of Binomial Distribution We now already know that binomial distribution gives the probability of a different set of outcomes. In real life, the concept of the binomial distribution is used for: Finding the quantity of raw and used materials while making a product. Taking a survey of positive and negative reviews from the public for any specific product or place. By using the YES/ NO survey To find the number of male and female students in a university. The number of votes collected by a candidate in an election is counted based on 0 or 1 probability. Consider a card selected at a random and replaced. If this experiment is repeated 5 times, let us find the probability of selecting exactly 3 hearts. Let us determine the number of trials, success, and the failure. The trial is the drawing a card 5 times. Thus n = 5. success: card drawn is a heart = p = 1/4 = 0.25 failure: card drawn is not a heart = q = 1-0.25 = 0.75 Using the binomial distribution formula, we get 5C \(_3\) (0,25)3 (0.75)2 = 0.088 Binomial Distribution Mean and Variance For a binomial distribution, the mean, variance and standard deviation for the given number of success are represented using the formulas Mean, μ = np Variance, σ2 = npq Standard Deviation σ= √(npq) Where p is the probability of success q is the probability of failure, where q = 1-p Binomial Distribution Vs Normal Distribution The main difference between the binomial distribution and the normal distribution is that the binomial distribution is discrete, whereas the normal distribution is continuous. It means that the binomial distribution has a finite amount of events, whereas the normal distribution has an infinite number of events. In case, if the sample size for the binomial distribution is very large, then the distribution curve for the binomial distribution is similar to the normal distribution curve. Properties of Binomial Distribution The properties of the binomial distribution are: There are only two distinct possible outcomes: true/false, success/failure, yes/no. There is a fixed number of 'n' times repeated trials in a given experiment. The probability of success or failure remains constant for each attempt/trial. Only the successful attempts are calculated out of 'n' independent trials. Every trial is an independent trial on its own, this means that the outcome of one trial has no effect on the outcome of another trial. Important Notes on Binomial Distribution For using the binomial distribution, the number of observations or trials in an experiment is fixed or finite. Each observation/attempt/trial is independent on its own. This means none of the trials have an effect on the probability of the next trial. Each trial has an equal probability of occurrence. The probability of success is exactly the same from one trial to another. ☛ Related Articles: Normal Distribution Formula Cumulative Frequency Frequency Distribution Example 1: If a coin is tossed 5 times, using binomial distribution find the probability of: (a)Exactly 2 heads (b) At least 4 heads. Solution: (a) The repeated tossing of the coin is an example of a Bernoulli trial. According to the problem: Number of trials: n=5 Probability of head: p= 1/2 and hence the probability of tail, q =1/2 For exactly two heads: x=2 Using binomial distribution formula, P(x=2) = 5C2 p2 q5-2 = 5! / 2! 3! × (½)2× (½)3 P(x=2) = 5/16 (b) For at least four heads, x ≥ 4, P(x ≥ 4) = P(x = 4) + P(x=5) Hence, using binomial distribution formula, P(x = 4) = 5C4 p4 q5-4 = 5!/4! 1! × (½)4× (½)1 = 5/32 P(x = 5) = 5C5 p5 q5-5 = (½)5 = 1/32 Answer:Therefore, P(x ≥ 4) = 5/32 + 1/32 = 6/32 = 3/16 Example 2: For the same question given above, using the binomial distribution find the probability of getting at most 2 heads. Solution: Solution: P(at most 2 heads) = P(X ≤ 2) = P (X = 0) + P (X = 1) P(X = 0) = (½)5 = 1/32 Using binomial distribution formula, we get P(X=1) = 5C1 (½)5= 5/32 Answer: Therefore, P(X ≤ 2) = 1/32 + 5/32 = 3/16 Example 3: A random variable X has the following binomial distribution. Determine P(X>6) and P(0